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Innovative Teaching Mathematics through Lesson Study III
—Focusing on Mathematical Communication

Project Report, Chinese Taipei

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Abstract.

We present findings and reflections on lesson study and on mathematical communications along with video recordings of a 5th grade lecture on *Line Symmetry* taken place on May 19, 2008, in a small-town elementary school in Taiwan. The project is sponsored by Ministry of Education (MOE) and runs through the year of 2008. We noted some features and requisites for mathematical communications in the classrooms, and we found the format of lesson study helpful and inspiring. We are looking forward to effectively improve the mathematics teaching in Taiwan by releasing the results and products of this project.

1. Backgrounds

The project was proposed to MOE after the APEC meeting in Tokyo, 2007. It was approved soon, and the team is supported through the year of 2008. The principal investigator is Prof. Lin, Chang-Shou, member of Academia Sinica and faculty of National Taiwan Univ. The team includes three elementary school teachers Chen, Jun-Yu (Ji-An E. S., Hua-Lian), Lin, Shu-Jun (Dong-Men E. S., Taipei) and Zhang, Lin-Wei (Dao-Hsiang E. S., Hua-Lian), and two mathematicians Ong, Ping-Zen (National Taiwan Univ.) and Shann, Wei-Chang (National Central Univ.).

The project starts in January, 2008, with a discussion of lecture targets and topics. Teachers proposed their lecture plans in March. Meetings are held in April to discuss on the plans similar to the format and spirits of *lesson study*. Four lectures are actually given to two classes by two teachers in May. All four lectures are fully digitally recorded by three professional cameras with all investigators on site. The edited digital videos are ready in June, and one (on the line of symmetry) was chosen to be presented in APEC-KKU symposium. The other three recordings are also to be finished with discussions and notes. The scripts of the presented lecture are written in Chinese and translated into English in July. To conclude the project, discussions and suggestions on the four lectures, along with edited video files, will be released on a

website and the ideas will be delivered to elementary school teachers through various channels.

The lecture presented here took place in the morning of Monday, May 19, 2008. The lecture was given to an ordinary class of co-ed 5^{th} grade pupils by a male math teacher with 10 years of experience. This teacher is not the "mentor" of the class, he meets the class four times a week. More than one half of the students are native residents, they are from families in the middle or lower social classes. The students were not prepared nor anticipated for the lecture.

Before this lecture, students have basic ideas about angles, right angles, perpendicular, parallel, triangles, and quadrilaterals. Students can use Latin alphabets to label vertices and line segments.

This was the second lecture in the series of Line Symmetry. In the previous lecture, the class learned to recognize the line symmetry of regular triangles, isosceles triangles, squares, and rectangles. They know how to find the line of symmetry of these figures by folding paper cards.

2. Goals of the Lecture

Starting with an isosceles triangle, and then a rectangle, that they have been playing with in the previous lecture, students recognize the corresponding points and corresponding lines with respect to the line of symmetry. Next, students acquire one property of the line of symmetry. That is, it orthogonally bisects the line segment of a pair of corresponding points. With this property, students are asked to complete the missing half of a line symmetric figure. The activities start with dots and lines and progress to a Christmas tree.

3. Findings

On the communications in general, we noticed interesting characteristics of classroom communications. It might be true that each class has one kind of another of this subculture characteristics. For instance, the teacher can effectively re-collect the class's attention by claiming "Seat Posture One." [19:05—19:15. This referrs to the digital video file, 19:05 means the temporal position of 19 minuts and 5 seconds.] And when the teacher announced "Last Five Seconds," students who are done with the job will chant together the count-down (which was a lot faster than the true five seconds) [18:59—19:07].

When we are running short of time, we tend to be in a hurry and ignore others' opinions. This happens in the classroom too, as is observed in this lecture [35:10—35:30]. This is hard to avoid, but we should encourage ourselves to take advantage of such inspiring moments and allow the lecture to go different ways. It would be even better if teachers are prepared with such situations, so they know

exactly how to respond. A *lesson study* prior to the lecture would be helpful.

On the mathematical communications in particular, we think the lecture could be more impressive to the students if they were aware of the purpose for naming the corresponding points. It just might be for the communication of "which" distances are equal? (The distance from each of the corresponding points to the line of symmetry.) Without the language, teacher and students can only make visual communications.

Secondly, the instructor would be doing a better job if he follows up the purpose for noticing the orthogonality. It just might be for the communication of a strategy to determine the corresponding point with respect to the line of symmetry: Draw a line that is perpendicular to the line of symmetry, and determine the distance by counting the cells [06:30—10:40].

Finally, when the activities were successfully done, did students understand what they are doing? What was the purpose for the determination of the corresponding point? It just might be for the communication of a strategy to draw the corresponding line: Determine the corresponding points of the endpoints of a line segment, then connect the points. Later on, some students claimed they drew the corresponding line "directly," which was only a disguise of the same strategy [37:00—38:00]. If the terms and strategies were made clear in the first place, there might not be such kind of mis-understanding.

When students claimed that they found the corresponding lines "directly," the instructor challenged them to determine the slope of the corresponding line. Both of them cannot express the idea precisely, because they are short of the exact mathematics terminology (and therefore lack of a certain mathematical concept). [34:45—35:05]

During the entire lecture, we heard the confusion of common and mathematical meanings. For instance, The Chinese character 角 (jiao^3) means "angle" in the mathematical sense, while it means "horn" and "corner" in the common sense. Because of the later sense, many students also use it for the meaning of "vertex," as we heard in the lecture. The similar situation is for 邊 (bian), which means "edge" in the mathematical sense, while it means "boundary" or "on this/that/the near/the far side" in the daily use. When we talk about "here" or "there" in Chinese, we end up talking a lot of 邊. It is confusing when edges are involved.

4. Reflections

With the hands-on exercise through the procedure of Lesson Study, we think it shall be practical and beneficial for lecture preparations and teaching experiences sharing.

Although we have had four hours of discussion before hand, we did not expect some students might think they have drawn the corresponding lines “directly.” The teacher was not prepared to respond on that situation. More colleague observers shall be more helpful, before or after the lecture.

It might be true that each language has the same problem of confusing meanings in different ways. It is part of the mathematics education to help students use the correct meaning of characters or phrases in appropriate contexts.

When we have the face-to-face communication, it is a common practice to point to the objects and expressing oneself with “this” and “that,” “here” and “there.” However, we think in the classroom teachers can be more precise by choosing the terminologies carefully and professionally.

After all, why shall we care about mathematical communications in a classroom? Reflections on the observations of this lecture, we think the mathematical communication helps to focus on the key concepts of a lecture, to make clear the objectives of what we are learning in a lecture.